

Curing Singularities: From the Big Bang to Black Holes

Janna Levin

Center for Particle Astrophysics, UC Berkeley, Berkeley, CA 94720-7304

Singular spacetimes are a natural prediction of Einstein's theory. Most memorable are the singular centers of black holes and the big bang. However, dilatonic extensions of Einstein's theory can support nonsingular spacetimes. The cosmological singularities can be avoided by dilaton driven inflation. Furthermore, a nonsingular black hole can be constructed in two dimensions.

The big bang and black holes are both spectacular predictions of Einstein's theory which share a singular nature. The singularities mark the breakdown of the classical theory, as predictability of the future or past is lost for world-lines which run into or out of the singularity. Infinite energy scales are invoked as the curvature invariants become infinite. The ultimate theory of quantum gravity, it is often hoped, will temper these singularities. Still, even without the full quantum theory, singularities can be avoided classically.

As argued by the Hawking-Penrose theorems, Einstein gravity plus any ordinary matter will spawn singular spacetimes [1]. The singularity theorems assume that both the Einstein equations hold and that the matter sector obeys the strong energy condition. For a diagonal energy-momentum tensor $T_\nu^\mu = (-\rho, p_1, p_2, p_3)$, the weak energy condition states

$$\rho \geq 0 \quad \text{and} \quad \rho + p_{1,2,3} \geq 0 , \quad (1.1)$$

while the strong energy condition states

$$\rho + p_1 + p_2 + p_3 \geq 0 \quad \text{and} \quad \rho + p_{1,2,3} \geq 0 . \quad (1.2)$$

(See for instance [2].) With the advent of inflation, violations of the strong energy condition are commonplace. The negative pressure needed to drive the inflationary growth of the universe can produce $p \leq -\rho$, though it is still advisable to respect the positivity of the energy density, $\rho > 0$. If the energy conditions are violated, the singularity theorems do not hold. While this does not ensure a nonsingular universe, many inflationary cosmologies can be shown to be nonsingular.

In general, simple conditions can be found on the matter stress tensor such that the minisuperspace of all homogeneous, isotropic cosmologies ($ds^2 = -dt^2 + a^2 d\ell^2$) is nonsingular. For this subset of all possible metrics, the curvature invariants depend only on H, \dot{H} and a where $H = \dot{a}/a$. An *initial* singularity will be avoided for an expanding universe if

$$\dot{H} = -\frac{4\pi}{M_{PL}^2} (\rho + p) \geq 0 ; \quad (1.3)$$

that is, if $p \leq -\rho$ and the energy conditions on which the singularity theorems are based are violated. Eqn (1.3) also ensures that null geodesics do not converge, $R_{\mu\nu} n^\mu n^\nu \propto -\dot{H} \leq 0$ indicating geodesic completeness [4]. While eqn (1.3) skirts an initial singularity, there may still be a *future* singularity, particularly if the model is superinflationary, i.e., $p < -\rho$ and so $\dot{H} > 0$. To avoid the future singularity, the evolution should roll over to

$$\text{sgn}(H)\dot{H} = -\text{sgn}(H)\frac{4\pi}{M_{PL}^2} (\rho + p) \leq 0 \quad (1.4)$$

before the singularity strikes. The sign of H incorporates the possibility that the universe contracts. For an initially expanding universe, condition (1.4) requires H

grow faster than or equal to a constant. As we look back in time, H decreases and therefore must have been less than infinite. Similarly a grows faster than or equal to e^t and so is finite as $t \rightarrow 0$. If we push time back to $-\infty$, then the space can be continued onto an initially large and contracting universe. Condition (1.4) then requires that $\dot{H} < 0$. In the past, H gets less negative and so must have been finite.

It may still be possible to find spacetimes which meet the above criteria but are singular in higher derivatives of H [3]. Protection against singularities in higher derivatives of H requires $\text{sgn}(d^n H/dt^n) d^{n+1} H/dt^{n+1}$ positive in the past and negative in the future. The curvature invariants depend also on \dot{H} . If they are to be finite, the equation of state for matter is restricted by $|p| < \infty \rho$, a reasonable requirement. In short, if matter obeys an equation of state bounded by $-\infty < p < 0$ in the past and $0 < p < \infty$ in the future, then the spacetime should be nonsingular.

The maximally symmetric de Sitter cosmology is the simplest nonsingular, inflationary universe. The energy momentum tensor is provided by a cosmological constant Λ with $\rho_\Lambda = -p_\Lambda$. In standard inflationary models these nonsingular conditions are difficult to maintain [4]. All energy densities which scale as $\rho \sim a^{-\alpha}$ will dominate over any constant cosmological density as the scale factor goes to zero, thereby reinstating a singularity. A generic prescription for avoiding precisely this problem has been based on de Sitter regularity [5]. The prescription is for a limiting curvature with a kind of asymptotic freedom. The coupling between gravity and matter becomes weaker as the limiting curvature is reached and de Sitter evolution reigns. Superinflationary models may be more robust than pure de Sitter. For these, the inflationary component becomes more effective as the would be singularity is approached.

There are very few forms of matter which induce negative pressures. Again, there is the celebrated cosmological constant. Less well known, a gas of fundamental strings can have negative pressure [6]. Before the advent of the now standard inflationary cosmology, Starobinsky's \mathcal{R}^2 inflation was first advanced as a nonsingular big bang [7]. Another modification to Einstein's theory, motivated both by ordinary quantum corrections and string theories, is generalized dilaton-gravity. Classes of such models have been found to generate a negative pressure [8,9]. Recently nonsingular dilaton cosmologies were selected on the basis of their kinetic coupling [10]. It can be shown that all of these kinetic couplings lead to negative pressures and kinetic inflation. These models are ideal for satisfying the conditions (1.3)-(1.4) since they can begin superinflationary and then connect onto a regular decelerating cosmology [8].

To illustrate how the dilaton avoids singularities consider the theory of gravity

$$A = \int d^4x \sqrt{-g} \left[-\varphi \mathcal{R} + \frac{\omega}{\varphi} (\partial\varphi)^2 \right] . \quad (1.5)$$

The lowest energy effective action from string theory predicts $\omega = -1$. The string theory dilaton leads to superinflation and begins nonsingular. In fact, the cosmos begins asymptotically flat [9]. However, it quickly runs into a problematic future singularity. In general, higher order string contributions can generate a variable $\omega(\varphi)$ [11]. Regardless of motivation, $\omega(\varphi)$ may be chosen such that the universe is nonsingular in both the past and the future [8,10].

Consider for the sake of simplicity $\omega = -4/3$. The evolution then appears identical to a de Sitter spacetime. The kinetic energy in the Planck field is

$$\rho_\varphi/\varphi \propto \left(\frac{\dot{\varphi}}{\varphi}\right)^2 = \text{constant} . \quad (1.6)$$

Then $H^2 = \text{constant}$ and $a \sim e^t$ while $\varphi \sim e^{-3t}$. All of the curvature invariants are de Sitter and nonsingular. A simple test particle moving along the space has no means by which to distinguish this from de Sitter.

While the curvature invariant is finite and in this sense the space is nonsingular, there are still some subtleties. All models of the form (1.5) are degenerately conformal to a singular universe described by Einstein gravity plus a minimally coupled scalar field. However, there is no paradox since the conformal transformation connecting the two is singular. Thus the singular Einstein space is only conformal to a portion of the nonsingular dilaton space. An analogous relationship arises with the common tool of transforming from expanding to comoving coordinates. A singular expanding space is conformally transformed to a nonsingular, comoving Minkowski space such that the conformal transformation carries the singularity. (Having said this however, gravity wave probes may be sensitive to the behaviour as $t \rightarrow -\infty$ as argued in Ref. [12]. If this is so, the space while bearing finite curvature invariants is not stably nonsingular.)

The limiting curvature hypothesis was used in $(1+1)$ dimensional gravity to locate nonsingular $2D$ black holes [5]. A potential was chosen to support the dilaton and the spacetime in a nonsingular configuration. * Since those solutions are also dilaton motivated it is of interest to show how a choice of the kinetic coupling can accomplish the same task. A dilaton-metric configuration similar to that of Ref. [5] is found below which is supported by an astute choice of the kinetic coupling.

The action is the two dimensional version of (1.5). A metric of the form

$$ds^2 = -h(r)dt^2 + h^{-1}(r)dr^2 \quad (1.7)$$

is sought. The trace of $G_{\mu\nu} = T_{\mu\nu} = 0$ leads to

$$\square\Phi = 0 \quad \Rightarrow \quad \varphi'' = 0 . \quad (1.8)$$

The constants of integration are chosen such that $\varphi = r$.

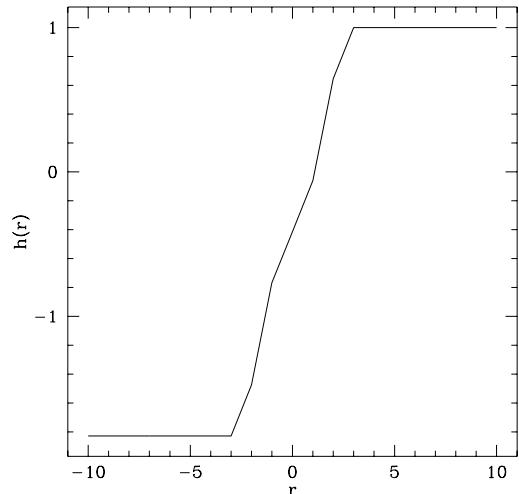


FIG. 1. The metric component $h(r)$.

The Einstein Equations require

$$h' = \frac{\omega}{\varphi} , \quad (1.9)$$

and the only curvature invariant is

$$\mathcal{R} = \left(\frac{\omega}{\varphi}\right)' = h(r)'' . \quad (1.10)$$

The behaviour desired is for the metric to behave as a normal Schwarzschild solution far from the horizon;

$$\lim_{r \rightarrow \infty} h(r) \rightarrow \left(1 - \frac{2m}{r}\right) . \quad (1.11)$$

Nonsingularity requires

$$\lim_{r \rightarrow 0} \mathcal{R} \rightarrow \text{finite.} \quad (1.12)$$

An example of a kinetic coupling, ω , which satisfies both (1.11) and (1.12) is

$$\frac{\omega}{\varphi} = \frac{2m\varphi^2}{\varphi^4 + m^4} . \quad (1.13)$$

Eqn (1.9) can be integrated and is depicted in Fig. 1. Notice, it passes through zero but is never infinite. There is a Killing vector $\xi = (h, 0)$ which becomes null at $h(r) = 0$ signaling the occurrence of an event horizon. Consequently, photons emitted from that point are infinitely redshifted and the surface is black.

The curvature invariant

$$\mathcal{R} = -\frac{4mr(r^4 - m^4)}{(r^4 + m^4)^2} \quad (1.14)$$

* The potential chosen in Ref. [5] may actually permit a singularity if φ is allowed negative values.

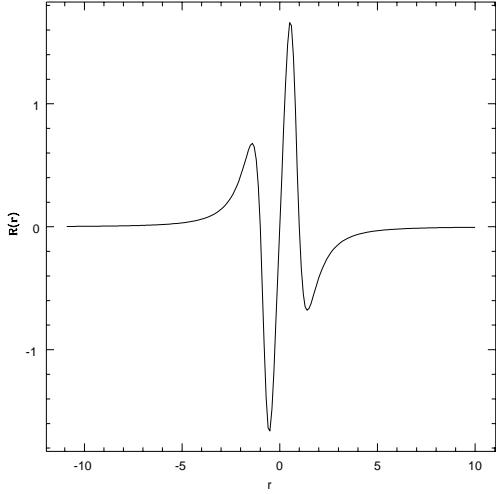


FIG. 2. The nonsingular Ricci scalar.

is zero as $r \rightarrow \infty$ and zero as $r \rightarrow 0$. It is nowhere singular as shown in Fig. 2.

This spacetime appears geodesically complete also as can be seen with the Lagrangian,

$$\mathcal{L} = \frac{1}{2} h \dot{t}^2 - \frac{1}{2} h^{-1} \dot{r}^2 \quad (1.15)$$

where an overdot denotes differentiation with respect to an affine parameter λ . The metric is static so that $h \dot{t} = E$, which is chosen to be $E \equiv 1$. Null geodesics lie on $h \dot{t}^2 - h^{-1} \dot{r}^2 = 0$ so that $\dot{r}^2 = 1$. Combine the two geodesic equations to find

$$\left(\frac{dr}{dt} \right)^2 = h^2(r) \quad (1.16)$$

Notice that

$$\frac{dr^2}{dt^2} = -h' h \quad \lim_{r \rightarrow 0} \rightarrow 0. \quad (1.17)$$

There is no force at the center, unlike a singular black hole for which the acceleration is infinitely negative driving the photon out the cut in spacetime.

At large r this has the features of a black hole. At some finite r there is a surface of infinite redshift and small curvature. Inside, the curvature rises to some maximum value but then turns over and vanishes at the center. The event horizon is only one point as shown in Fig. 3. Once inside the event horizon, photons move forever along the axis to an asymptotically flat space. It is sensible that there are no truly trapped surfaces. If there were, photons would have nowhere to go but out a singularity. For this reason, a 4D black hole would have to grow an infinitely long throat or perhaps even a nested expanding universe [13].

While this has all of the features of a black hole there is still some ambiguity in the interpretation of the spacetime. In particular, in 2D, the space can be conformally

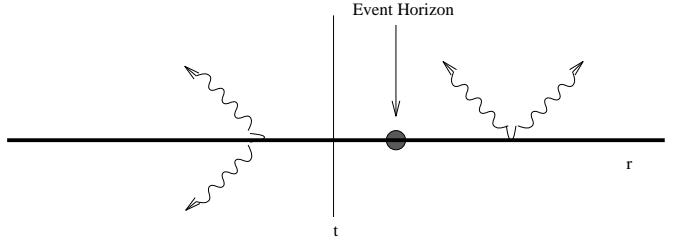


FIG. 3. A schematic spacetime diagram of the black hole or rather, the black point. Photons emitted from the point indicated are infinitely redshift to the right. Light which is drawn through the event horizon moves forever to the left as represented by the tilting of the light cones.

mapped by $g_{\mu\nu} = \exp(-4T(\varphi))g_{\mu\nu}^T$ onto an everywhere flat spacetime with the action

$$A = \int d^2x \sqrt{-g_T} [-\varphi \mathcal{R}_T] \quad (1.18)$$

There is no kinetic coupling to choose. Variation with respect to φ gives $\mathcal{R}_T = 0$ everywhere. The transformation

$$T(\varphi) = 2 \int \frac{dr}{h'} = \frac{r^3}{3m} - \frac{m^3}{r} \quad (1.19)$$

is singular both at 0 and ∞ . However, any observer who measures the wavelength of a photon emitted from near the black surface will perceive an infinite redshift. Observers believe they live in a flat space but that their rulers are warped throughout space by the coupling to the field φ . When the observer measures the wavelength of a photon, the ratio of the wavelength to the ruler length is a conformal invariant. In this sense, the same point appears dark in both frames.

Classical dynamics can render both a universe and 2D black holes nonsingular. The interiors of nonsingular black holes are even able to support inflation, thus the destructive singularity is avoided in favor of creating a universe [14,5]. Not only can singularities be avoided but so too can all quantum gravity scales. While nonsingular solutions are noteworthy, it is fair to say that there is as yet nothing natural about these singularity avoidance mechanisms. Further they may not be stable [4,12]. To put it another way, they may not be attractors in the space of all possible solutions. Still, the possibility of a nonsingular cosmos offers an interesting sketch of the early universe. The violent energetic beginning is replaced with a smooth classical state. Creation of such a nonsingular universe from nothing may be less costly. Inflation then transports the cosmos into a big bang epoch which culminates in a universe vast and energetic enough for us to witness.

I am grateful to J.R. Bond, A. Börde, R. Brandenberger, P. Ferreira, N. Kaloper, and K. Rama for valuable

contributions to these ideas. This research is supported in part by a President's Postdoctoral Fellowship.

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